Concepts of Probability.

- **Probability** is a field of mathematics that deals with events and quantifies their likelihood of occurring with numerical values ranging from 0 to 1. Higher probabilities indicate a greater chance of the event happening. It is mainly a ratio between the given event and the total number of events.
- Probability is defined as the likelihood of the occurrence of any event. Probability is expressed as a number between 0 and 1, where, 0 is the probability of an impossible event and 1 is the probability of a sure event.

Examples of probability.

1. **Basic probability:**

   a. Flipping a coin: Probability of heads is 1/2, probability of tails is 1/2.

   b. Rolling a die: Probability of getting a 6 is 1/6.

   c. Drawing a card from a deck: Probability of drawing a king is 4/52 (or 1/13).

2. **Conditional probability:**
   - What is the probability of drawing a face card from a deck given that you already drew a heart?

3. **Independent events:**
   - Flipping a coin twice and calculating the probability of getting heads on both flips.

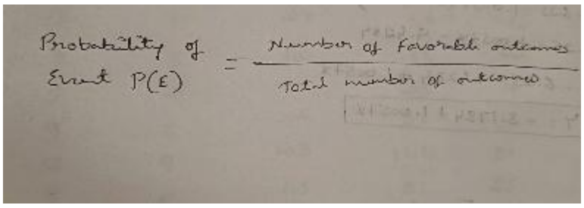
4. **Real-world applications:**
   - Weather forecasting: Predicting the chance of rain tomorrow.
   - Medical research: Determining the likelihood of a patient developing a disease based on certain factors.
   - Market research: Estimating the probability of a new product being successful.

Probability of an Event

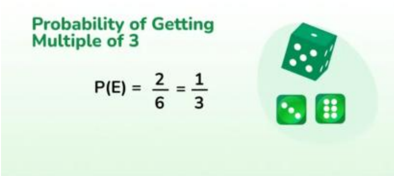
The probability of an event E, denoted by P(E), is a number between 0 and 1 that represents the likelihood of E occurring.

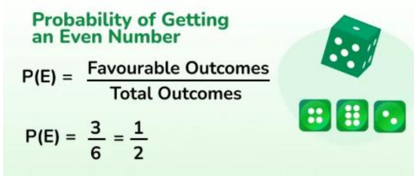
1. If P(E) = 0, the event E is impossible.
2. If P(E) = 1, the event E is certain to occur.
3. If 0 < P(E) < 1, the event E is possible but not guaranteed.

Formula for Probability



Examples - 1





Note: The sum of the probabilities of all events in a sample space is always equal to 1.

Example 2 - When we toss a coin, there are only two possible outcomes: Heads (H) or Tails (T). However, if we toss two coins simultaneously, there will be four possible outcomes: (H, H), (H, T), (T, H), and (T, T).

Sample Space & Event Sample Space

- **Sample Space**: The sample space, often denoted by S, is the set of all possible outcomes of an experiment. For example, when rolling a six-sided die, the sample space is S = {1, 2, 3, 4, 5, 6}.
- **Event**: An event is any subset of the sample space. It represents a specific outcome or a combination of outcomes. There are many different types of events in Probability such as Impossible and Sure Events, Mutually Exclusive Events, Exhaustive Events, Dependent and Independent Events etc. **For example,** rolling an even number E = {2, 4, 6}) is an event in the context of rolling a die.

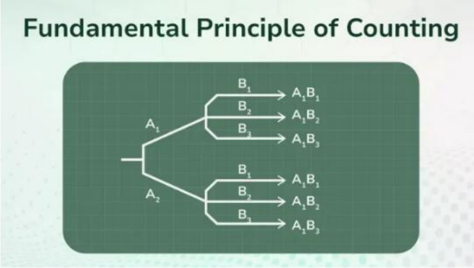
Applications of Probability

Some of the common events which we can use applications of probability to check the results are:

1. Choosing a card from the deck of cards
2. Flipping a coin
3. Throwing a dice in the air
4. Pulling a red ball out of a bucket of red and white balls
5. Winning a lucky draw

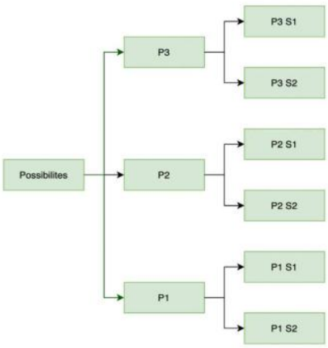
Fundamental Principle of Counting

1. Fundamental Principle of Counting is the basic principle that helps us to count large numbers in a non-tedious way.
2. Fundamental Principle of Counting is very helpful in finding all the possible combinations in a particular situation.



Example 1 - Say a person has 3 pairs of pants and 2 shirts and a question pops up, how many different ways are there in which can he dress? There are three different ways of choosing pants as there are three types of pants available. Similarly, there are two ways of choosing shirts.

Let's see all the different ways of dressing through a diagram. Considering P1, P2, and P3 as pants and S1, S2 as shirts. The tree is given below lists the range of possibilities.



Example 2 - Count the number of possibilities when a coin is tossed 3 times.
A coin toss can have two outcomes, either Heads(H) or Tails(T), and in case of tossing three coins simultaneously the total number of ways in which this can happen is,
= Total Outcome of First Coin Toss × Total Outcome of Second Coin Toss × Total Outcome of Third Coin Toss
= 2 × 2 × 2
= 8
Thus, tossing three coins simultaneously can have 8 different possible outcomes.

The fundamental principle of counting is studied under two headings that include

1. Addition Rule - Addition Rule states that for two possible events A and B where A and B both are mutually exclusive events, i.e. they have no outcome in common and if event E is defined as occurring in either event A or event B then the possible number of ways in which event E can occur is, Where n(A), n(B), and n(E) are the number of events of A, B, and E respectively.

$$n(E) = n(A) + n(B)$$

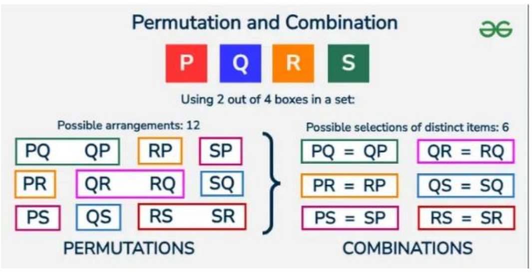
2. Multiplication Rule - Multiplication Rule states that for "n" mutually independent events, P1, P2, P3, …Pn. The number of in which these events can occur is n(P1), n(P2), n (P3) … n(Pn) respectively. Now we define an event E such that it is happening all the events simultaneously then the number of ways this can happen is This is called the multiplication rule of the Fundamental Principle of Counting.

$$n(E) = n(P_1) \times n(P_2) \ldots\ldots\ldots \times n(P_n)$$

Permutations and Combinations

Permutations and Combinations are the most fundamental concepts in mathematics related to picking items from a group or set.

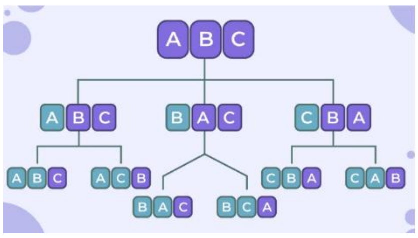
- Permutation is arranging items considering order of selection from a certain group.
- Combination is selecting items without considering order.
- For example, in the below diagram, PQ and QP are different in permutation but same in combination. Therefore, we have more permutations than combinations.



Permutation Meaning

Permutation is the distinct interpretations of a provided number of components carried one by one, or some, or all at a time. For example, if we have two components A and B, then there are two likely performances, AB and BA.

A numeral of permutations when ‘r’ components are positioned out of a total of ‘n’ components is n Pr. For example, let n = 3 (A, B, and C) and r = 2 (All permutations of size 2). Then there are 3P2 such permutations, which is equal to 6. These six permutations are AB, AC, BA, BC, CA, and CB. The six permutations of A, B, and C taken three at a time are shown in the image added below:



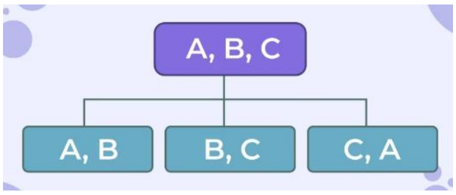
Permutation Formula - is used to find the number of ways to pick r things out of n different things in a specific order and replacement is not allowed and is given as follows

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination Meaning

It is the distinct sections of a shared number of components carried one by one, or some, or all at a time. For example, if there are two components A and B, then there is only one way to select two things, select both of them.

For example, let n = 3 (A, B, and C) and r = 2 (All combinations of size 2). Then there are 3C2 such combinations, which is equal to 3. These three combinations are AB, AC, and BC.



Note: In the same example, we have distinct points for permutation and combination. For, AB and BA are two distinct items i.e., two distinct permutations, but for selecting, AB and BA are the same i.e., same combination.

Combination Formula **-** is used to choose ‘r’ components out of a total number of ‘n’ components, and is given by:

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$^{n}C_{r} = \frac{n!}{r! \times (n-r)!} = \frac{^{n}P_{r}}{r!}$$

Difference Between Permutation and Combination

Permutation	Combination
In Permutation order of arrangement is important. For example, AB and BA are different combinations.	In Combination order of arrangement is not important. For example, AB and BA are the same combinations.
A permutation is used when different kinds of things are to be sorted or arranged.	Combinations are used when the same kind of things are to be sorted.
Permutation of two things out of three given things a, b, c is ab, ba, bc, cb, ac, ca.	The combination of two things from three given things a, b, c is ab, bc, ca.
Formula for permutation - $^nP_r = \frac{n!}{(n-r)!}$	Formula for Combination - $^nC_r = \frac{n!}{r!(n-r)!}$

Bayes' Theorem

- Bayes' Theorem is a mathematical formula that helps determine the conditional probability of an event based on prior knowledge and new evidence.
- It adjusts probabilities when new information comes in and helps make better decisions in uncertain situations.
- It provides a way to update the probability of a hypothesis based on new evidence. Mathematically, it is expressed as:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Formula for the Bayes theorem

Where,

- P(A) and P(B) are the probabilities of events A and B also P(B) is never equal to zero,
- P(A|B) is the probability of event A when event B happens,
- P(B|A) is the probability of event B when A happens.

Bayes Theorem Statement

Bayes Theorem for n set of events is defined as,
Let E1, E2…, En be a set of events associated with the sample space S, in which all the events E1, E2…, En have a non-zero probability of occurrence. All the events E1, E2…, E form a partition of S. Let A be an event from space S for which we have to find probability, then according to Bayes theorem,

$$P(E_i|A) = P(E_i)P(A|E_i) / \sum P(E_k)P(A|E_k)$$

for k = 1, 2, 3, …., n

Bayes Theorem Derivation

The proof of Bayes Theorem is given as, according to the conditional probability formula,

$P(E_i|A) = P(E_i \cap A) / P(A)$.....(i)

Then, by using the multiplication rule of probability, we get

$P(E_i \cap A) = P(E_i)P(A|E_i)$......(ii)

Now, by the total probability theorem,

$P(A) = \sum P(E_k)P(A|E_k)$.....(iii)

Substituting the value of $P(E_i \cap A)$ and $P(A)$ from eq (ii) and eq(iii) in eq(i) we get,

$$P(E_i | A) = P(E_i)P(A | E_i) / \sum P(E_k)P(A | E_k)$$

Bayes' theorem is also known as the formula for the probability of "causes". As we know, the E_i's are a partition of the sample space S, and at any given time only one of the events E_i occurs. Thus we conclude that the Bayes theorem formula gives the probability of a particular E_i, given the event A has occurred.

Bayes Theorem and Conditional Probability

- Bayes theorem (also known as the Bayes Rule or Bayes Law) is used to determine the conditional probability of event A when event B has already occurred.
- The general statement of Bayes' theorem is "The conditional probability of an event A, given the occurrence of another event B, is equal to the product of the event of B, given A and the probability of A divided by the probability of event B."

Terms Related to Bayes Theorem

1. Hypotheses
    - Hypotheses refer to possible events or outcomes in the sample space, they are denoted as E1, E2, …, En.

- Each hypothesis represents a distinct scenario that could explain an observed event.

2. Priori Probability
    - Priori Probability P(Ei) is the initial probability of an event occurring before any new data is taken into account.
    - It reflects existing knowledge or assumptions about the event.
    - Example: The probability of a person having a disease before taking a test.
3. Posterior Probability
    - Posterior probability (P(Ei|A) is the updated probability of an event after considering new information.
    - It is derived using Bayes Theorem.
    - Example: The probability of having a disease given a positive test result.
4. Conditional Probability
    - The probability of an event A based on the occurrence of another event B is termed conditional Probability.
    - It is denoted as P(A|B) and represents the probability of A when event B has already happened.
5. Joint Probability
    - When the probability of two more events occurring together and at the same time is measured it is marked as Joint Probability.
    - For two events A and B, it is denoted by joint probability is denoted as, P(A∩B).
6. Random Variables
    - Real-valued variables whose possible values are determined by random experiments are called random variables.
    - The probability of finding such variables is the experimental probability.

Bayes Theorem Applications

Bayesian inference is very important and has found application in various activities, including medicine, science, philosophy, engineering, sports, law, etc., and Bayesian inference is directly derived from Bayes theorem.
Some of the Key Applications are:

- Medical Testing → Finding the real probability of having a disease after a positive test.
- Spam Filters → Checking if an email is spam based on keywords.
- Weather Prediction → Updating the chance of rain based on new data.
- AI & Machine Learning → Used in Naïve Bayes classifiers to predict outcomes.

Difference Between Conditional Probability and Bayes Theorem

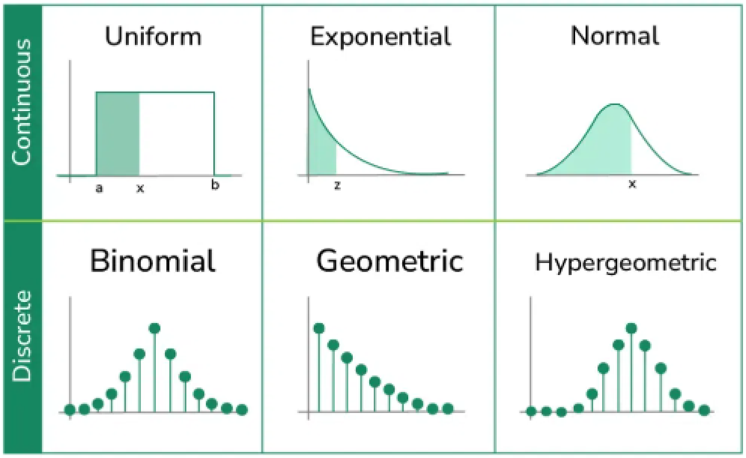
Bayes Theorem	Conditional Probability
Bayes Theorem is derived using the definition of conditional probability. It is used to find the reverse probability.	Conditional Probability is the probability of event A when event B has already occurred.
Formula: $P(A\|B) = [P(B\|A)P(A)] / P(B)$	**Formula:** $P(A\|B) = P(A \cap B) / P(B)$

Concept of Probability Distribution

- A probability distribution describes how the probabilities of different outcomes are assigned to the possible values of a random variable. It provides a way of modelling the likelihood of each outcome in a random experiment.
- **Probability Distribution** refers to the function that gives the probability of all possible values of a random variable. It shows how the probabilities are assigned to the different possible values of the random variable.
- A probability distribution is a statistical function that describes all the possible values and probabilities for a random variable within a given range.

Types of Probability Distribution.

1. Discrete Probability Distributions
2. Continuous Probability Distributions



In a **discrete probability distribution**, the random variable takes distinct values (like the outcome of rolling a die).

In a **continuous probability distribution**, the random variable can take any value within a certain range (like the height of a person).

Key properties of a probability distribution include:

- The probability of each outcome is greater than or equal to zero or 0 means it's impossible.
- The sum of the probabilities of all possible outcomes equals 1or 1 means it's certain.

Theoretical Probability Distributions or Probability Distribution Function.

Probability Distribution Function is defined as the function that is used to express the distribution of a probability. Different types of probability, are expressed differently. These functions are also used for Probability Density Functions for different variables.

For example, if we toss a fair coin, the probability of getting a head is 1/2. If we toss it for 50 times, the probability of getting a head is 25. We call this as the theoretical or expected frequency of the heads. But actually, by tossing a coin, we may get 25, 30 or 35 heads which we call as the observed frequency.

Types of Theoretical Distribution

1. Binomial Distribution
2. Poisson distribution
3. Normal distribution or Expected Frequency distribution

Binomial Distribution

Binomial Distribution is a probability distribution used to model the number of successes in a fixed number of independent trials, where each trial has only two possible outcomes: success or failure.

A random experiment whose outcomes are of two types namely success S and failure F, occurring with probabilities p and q respectively, is called a Bernoulli trial.

This distribution is useful for calculating the probability of a specific number of successes in scenarios like flipping coins, quality control, or survey predictions.

A Binomial Distribution for a random variable X = 0, 1, 2…, n is defined as the probability of success or failure in a series of independent trials. Each trial is independent of the others, and the distribution helps calculate the probability of various outcomes in these trials.

Conditions for Binomial Distribution

1. **Fixed Number of Trials**: There are a set number of trials or experiments (denoted by n), such as flipping a coin 10 times.
2. **Two Possible Outcomes**: Each trial has only two possible outcomes, often labeled as "success" and "failure." For example, getting heads or tails in a coin flip.
3. **Independent Trials**: The outcome of each trial is independent of the others, meaning the result of one trial does not affect the result of another.
4. **Constant Probability**: The probability of success (denoted by p) remains the same for each trial. For example, if you're flipping a fair coin, the probability of getting heads is always 0.5.

Binomial Distribution Formula

Let A be some event associated with a random experiment E, such that P(A) = p and P(A') = q = 1 – p. Assuming that p remains the same for all repetitions, if we consider n independent repetitions (or trials) of E and if the random variable (RV)X denotes the number of times the event A has occurred then X is called a **binomial random variable** with parameters n and p or we can say that X follows a **binomial distribution** with parameters n and p, or symbolically B(n, p). Obviously, the possible values that X can take, are 0, 1, 2…., n. Then the probability mass function of a binomial random variable is given by

Binominal Distributions

Mean = np

variance = npq

Standard deviation (σ) = $\sqrt{\text{variance}} = \sqrt{npq}$

$$P(X = r) = {}^{n}C_{r}\, q^{n-r}\, p^{r}$$

- **P (X = r)** is the probability of getting exactly r successes,
- **n** is the number of total trials,
- **r** is the number of successes in n Trials,
- **p** is the probability of success,
- **q** is the probability of failure, and
- **p + q = 1** and **r = 0, 1, 2, …, n**

Properties of Binomial distribution

- Binomial distribution is symmetrical if $p = q = 0.5$.
- It is skew symmetric if $p \neq q$.
- 4It is positively skewed if $p < 0.5$ and it is negatively skewed if $p > 0.5$
- For Binomial distribution, variance is less than mean
  Variance $npq = (np)q < np$

Binomial Distribution Calculation

Binomial Distribution in statistics is used to compute the probability of likelihood of an event using the above formula. To calculate the probability using binomial distribution we need to follow the following steps:

- **Step 1:** Find the number of trials and assign it as 'n'
- **Step 2:** Find the probability of success in each trial and assign it as 'p'

- **Step 3:** Find the probability of failure and assign it as q where q = 1-p
- **Step 4:** Find the random variable X = r for which we have to calculate the binomial distribution
- **Step 5:** Calculate the probability of Binomial Distribution for X = r using the Binomial Distribution Formula.

Poisson Distribution.

The Poisson distribution is a type of discrete probability distribution that calculates the likelihood of a certain number of events happening in a fixed time or space, assuming the events occur independently and at a constant rate.

It is characterized by a single parameter, **λ (lambda)**, which represents the average rate of occurrence of the event. The distribution is used when the events are rare and the number of occurrences is non-negative and can take on integer values (0, 1, 2, 3…).

The key assumptions of the Poisson distribution

1. Events occur independently of each other.
2. The average rate of occurrence (λ) is constant over the given interval.
3. The number of events can be any non-negative integer.

Poisson Distribution Formula

Formula

$$P(x = r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

Where,

- P(X = r) is the Probability of Observing k Events
- e is the Base of the Natural Logarithm (approximately 2.71828)
- λ is the Average Rate of Occurrence of Events
- r is the Number of Events that Occur

Poisson Distribution Table

This table is a tabulation of probabilities for a Poisson distribution and probabilities here can be calculated using the Probability Mass Function of Poisson Distribution which is given by PMF=

k (Number of Events)	P(X = k)
0	0.0498
1	0.1494
2	0 .2241
3	0.2241
4	0.1681
5	0.1009
6	0.0505
7	0.0214
2	0.0080
9	0.0027
10	0.0008

Poisson Distribution Characteristics

a. **Probability Mass Function (PMF)**: PMF describes the likelihood of observing a specific number of events in a fixed interval. It is given by: P(X = k) = (e-λ × λk) / k!

b. **Cumulative Distribution Function (CDF)**: CDF gives the probability that the random variable is less than or equal to a certain value. It is expressed as: F(x) = ∑(from k=0 to ⌊x⌋) (e-λ × λk) / k!

c. **Moment Generating Function (MGF)**: MGF provides a way to derive moments of the distribution. It is represented by: M(t) = $e^{(\lambda(e^\wedge t - 1))}$

d. **Characteristic Function (CF)**: CF is an alternative way to describe the distribution and is given by:

$$\phi(t) = e^{(\lambda(e^{\wedge}(it) - 1))}$$

e. **Probability Generating Function (PGF)**: PGF generates the probabilities of the distribution and is expressed as:

$$G(z) = e^{(\lambda(z - 1))}$$

f. **Median**: Median, which represents the central value, is approximately λ+ (1/3) − 0.02/λ.

g. **Mode**: Mode, or the most probable value, is simply the integer part of λ, denoted as ⌊λ⌋.

h. **Mean and Variance**: The mean (λ) and variance (λ) of a Poisson distribution are equal. This means that both the average number of events and the spread or variability around this average are characterized by the same parameter.

i. **Non-negative and Discrete**: The Poisson distribution describes the probability of non-negative integer values only, as it models counts of events. It is a discrete probability distribution.

j. **Memory lessness**: Events in a Poisson process are memoryless, meaning the probability of an event occurring in the future is independent of the past, given the current state. For example, if you're waiting for a bus, the probability of the bus arriving in the next minute doesn't depend on how long you've already been waiting.

k. **Independent Increments**: The number of events occurring in non-overlapping intervals is independent. For instance, if you're counting the number of cars passing through an intersection in one minute, the number of cars in the next minute is independent of the number in the previous minute.

l. **Rare Events Approximation**: When the average rate of occurrence (λ) is large and the probability of a single event is small, the Poisson distribution can approximate the binomial distribution. This is known as the "rare events" approximation, where the binomial distribution with a large number of trials and a small probability of success converges to a Poisson distribution.

m. **Skewness and Kurtosis**: Poisson distribution is positively skewed (skewness > 0) and leptokurtic (kurtosis > 0), meaning it has a longer tail on the right side and heavier tails than the normal distribution. However, for large values of λ, it becomes increasingly symmetric and bell-shaped, resembling a normal distribution.

Some other properties are:

- Poisson distribution has only one parameter "λ" where λ = np.
- Poisson distribution is positively skewed and leptokurtic.

Note: Here **leptokurtic** means values greater **kurtosis** than the normal distribution, and **kurtosis** is the nothing but the sharpness of the peak of the **frequency distribution curve**.

Difference between Binomial and Poisson Distribution		
Aspect	**Binomial Distribution**	**Poisson Distribution**
Nature	Discrete	Discrete
Number of Trials	Fixed (n)	Unlimited
Outcome	Success or Failure	Rare Events
Parameter	Probability of Success (p)	Average Event Rate (λ)
Possible Values	0 to n	0, 1, 2, . . .
Mean	$\mu = n \times p$	$\mu = \lambda$
Variance	$\sigma^2 = n \times p \times (1 - p)$	$\sigma^2 = \lambda$
Applicability	Limited to a fixed number of trials	Rare events over a large population
Example	Flipping a coin multiple times	Counting occurrences of an event
Assumptions	Independent trials, constant p	Rare events, low probability of success

Normal Distribution.

- Normal distribution, also known as the Gaussian distribution, is a continuous probability distribution that is symmetric about the mean, depicting that data near the mean are more frequent in occurrence than data far from the mean.
- Normal Distribution is defined as the probability density function of any continuous random variable for any given system. Now for defining Normal Distribution suppose we take f(x) as the probability density function for any random variable X.
- The curve traced by the upper values of the Normal Distribution is in the shape of a Bell, hence Normal Distribution is also called the "**Bell Curve**".

Key Features of Normal Distribution

- Symmetry: The normal distribution is symmetric around its mean. This means the left side of the distribution mirrors the right side.
- Mean, Median, and Mode: In a normal distribution, the mean, median, and mode are all equal and located at the centre of the distribution.
- Bell-shaped Curve: The curve is bell-shaped, indicating that most of the observations cluster around the central peak, and the probabilities for values further away from the mean taper off equally in both directions.
- Standard Deviation: The spread of the distribution is determined by the standard deviation. About 68% of the data falls within one standard deviation of the mean, 95% within two standard deviations, and 99.7% within three standard deviations.

Normal Distribution Examples

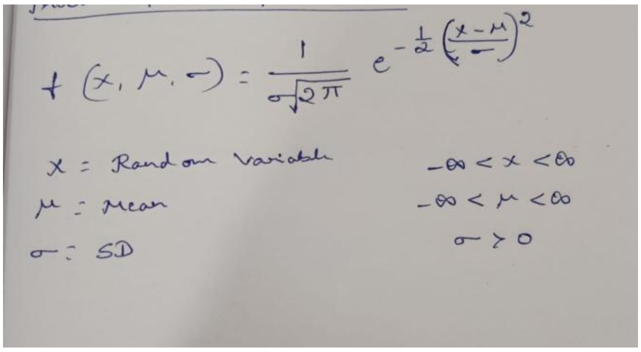
We can draw Normal Distribution for various types of data that include,

- Distribution of Height of People.
- Distribution of Errors in any Measurement.
- Distribution of Blood Pressure of any Patient, etc.

Properties of Normal Distribution.

a. The curve is symmetrical, unimodal and bell shaped.
b. All the values of y are greater than zero and approach zero as x approaches $\pm \infty$.
c. It can be proved that the area between the curve and the x-axis is one unit.
d. It can be proved that:
   (i) The mean, mode and median are all equal to the parameter μ.
   (ii) The standard deviation is equal to the parameter σ.
e. $P(x1 < x < x2)$ = area under the curve between the ordinates $\phi(x1)$ and $\phi(x2)$.
f. If $z = x - \mu / \sigma$, it can be proved that z has the same normal distribution for every pair of σ values of the parameters μ and σ.

Normal Distribution Formula – Probability Density Function (PDF)



Normal Distribution Curve

In any Normal Distribution, random variables are those variables that take unknown values related to the distribution and are generally bound by a range. An example of the random variable is, suppose take a distribution of the height of students in a class then the random variable can take any value in this case but is bound by a boundary of 2 ft to 6 ft, as it is generally forced physically.

- Range of any normal distribution can be infinite in this case we say that normal distribution is not bothered by its range. In this case, range is extended from $-\infty$ to $+\infty$.
- Bell Curve still exist, in that case, all the variables in that range are called Continuous variable and their distribution is called Normal Distribution as all the values are generally closed aligned to the mean value.

- The graph or the curve for the same is called the Normal Distribution Curve Or Normal Distribution Graph.

Normal Distribution Standard Deviation

Standard Deviation tells us how far the data is spread out from the mean value on either side. For smaller values of the standard deviation, the values in the graph come closer and the graph becomes narrower. While for higher values of the standard deviation the values in the graph are dispersed more and the graph becomes wider.

The normal distribution has a positive standard deviation and the standard deviation divides the area of the normal curve into smaller parts and each part defines the percentage of data that falls into a specific region This is called the Empirical Rule of Standard Deviation in Normal Distribution.

Formula for Standard Normal Variate.

Normal Variate is indicated by = X
Standard normal Variate is indicated by = Z

$$Z = \frac{x - \mu}{\sigma}$$

Characteristics of Standard Normal Distribution

Standard normal distribution is defined by the following characteristics:

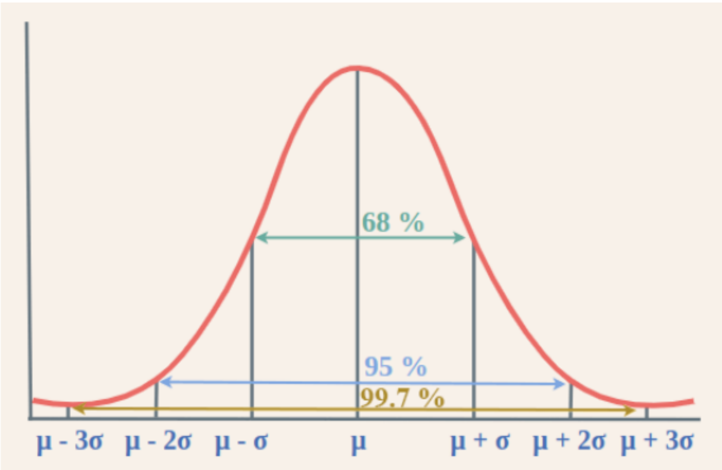
- **Mean**: The mean (average) is 0 that is symbolically represented as $\mu = 0$.
- **Standard Deviation**: The standard deviation is 1 that is symbolically represented as $\sigma = 1$.
- **Symmetry**: It is symmetric around the mean ($\mu = 0$).
- **Bell-Shaped Curve**: The graph is bell-shaped, that means most values cluster around the mean ($\mu = 0$).
- **Total Area Under Curve**: The total area under the curve is 1, representing the total probability.

- **68-95-99.7 Rule**: Approximately 68% of data falls within 1 standard deviation of the mean, 95% within 2 standard deviations, and 99.7% within 3 standard deviations.
- **Asymptotic**: The tails of the distribution approach, but never touch, the horizontal axis.
- **Unimodal**: It has a single peak at the mean ($\mu = 0$).
- **Standard Scores (Z-Scores)**: Any normal distribution can be transformed into the standard normal distribution using z-scores where $z = (x - \mu)/\sigma$.

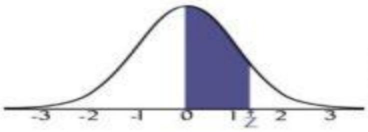
Empirical Rule states that,

- 68% of the data approximately fall within one standard deviation of the mean, i.e. it falls between {Mean – One Standard Deviation, and Mean + One Standard Deviation}
- 95% of the data approximately fall within two standard deviations of the mean, i.e. it falls between {Mean – Two Standard Deviation, and Mean + Two Standard Deviation}
- 99.7% of the data approximately fall within a third standard deviation of the mean, i.e. it falls between {Mean – Third Standard Deviation, and Mean + Third Standard Deviation}

Normal Distribution Graph



Studying the graph it is clear that using Empirical Rule we distribute data broadly in three parts. And thus, empirical rule is also called "68 – 95 – 99.7" rule.



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Importances of Normal distribution in Business Statistics.

- Normal distribution, also known as Gaussian distribution, is a bell-shaped curve that describes a large number of real-world phenomena. It's one of the most important concepts in statistics because it pops up in many areas of study.
- Bell-Shaped Curve: Imagine a symmetrical bell where the middle is the highest point and the tails taper off on either side. That's the basic shape of a normal distribution. Most data points cluster around the centre, and as you move further away from the centre, the data points become less frequent.
- Central Tendency: The centre of the bell curve represents the central tendency of the data, which means it shows where most of the values are

concentrated. This could be the mean, median, or mode, depending on the specific data set.

- Spread of Data: The width of the bell curve indicates how spread out the data is a wider curve means the data points are more dispersed, while a narrower curve signifies the data points are closer together.
- Random Variables: Normal distribution is typically used with continuous random variables, which can take on any value within a specific range. Examples include heights, weights, IQ scores, or exam grades.